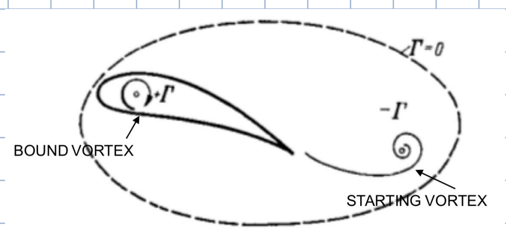
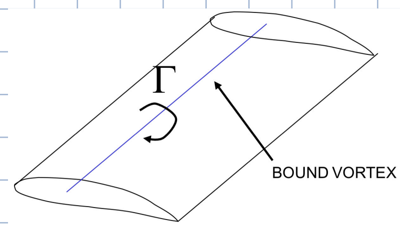
The Horseshoe Vortex :

Extends 2D single 'lumped vortex' model where Kutta condition was applied to a 3D finite wing:

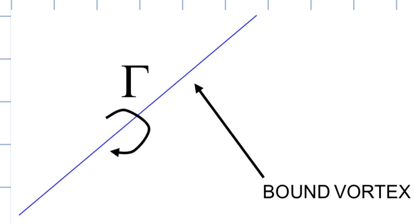
2D



3D



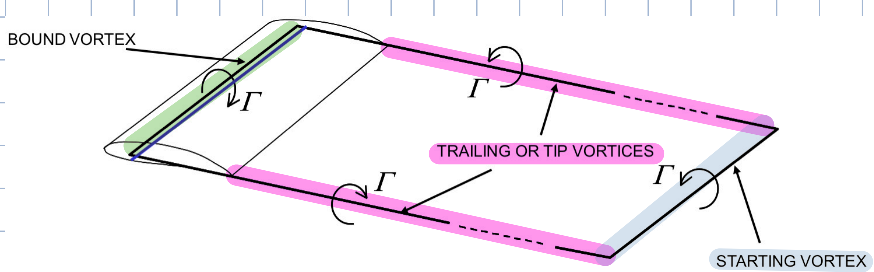
geometry does not exist in mathematical model, it just sees this:



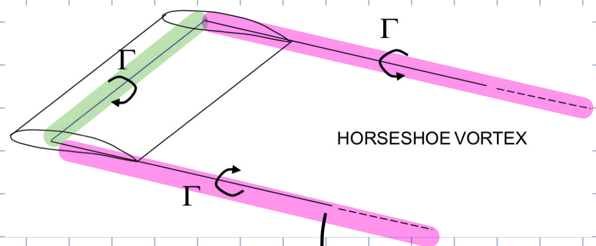
Helmholtz theorem says filament cannot end in fluid domain

continue as 'trailing' or 'tip' vortices based on knowledge of flow structure

Circuit 'completed' by starting vortex



Closed loop obeys Helmholtz law. (2)



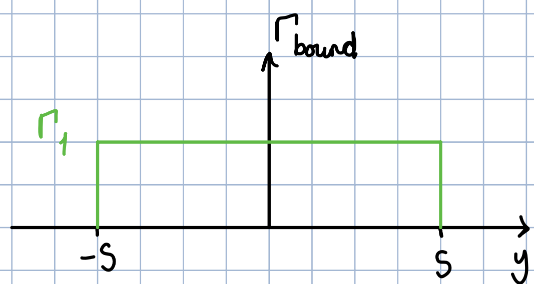
Vortex filaments induce velocities proportional to $\frac{1}{h}$ so as $h \uparrow$

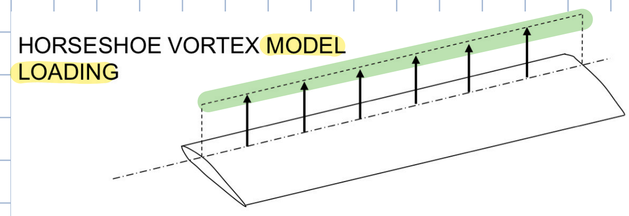
as the starting vortex moves downstream, this becomes negligible

semi-infinite trailing filaments

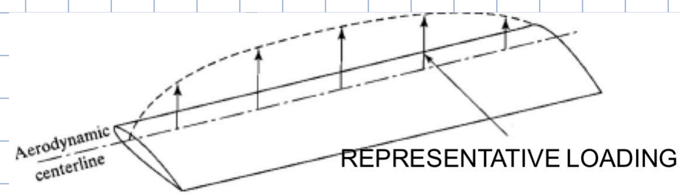
1^{st} Helmholtz law states that the strength of vortex filament is constant along length:

→ across bound vortex:





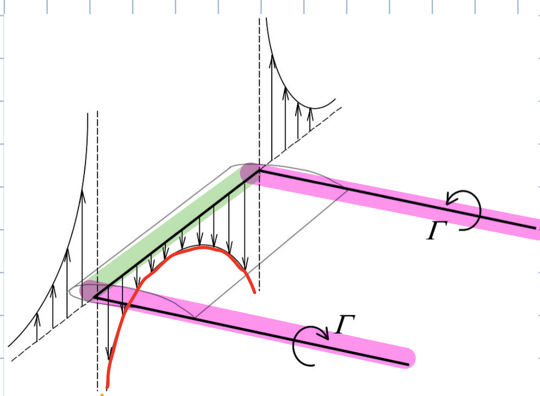
Compared with representative loading:



∴ Currently not a good representation

Bound (lifting) wing vortex does NOT induce a velocity on itself.

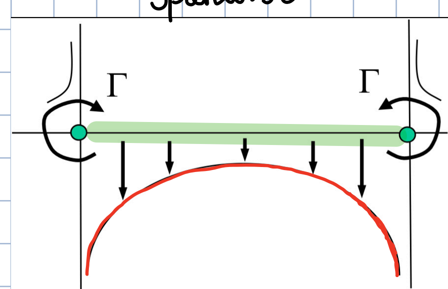
Trailing edge vortices DO induce velocity on bound vortex:



$-\infty$

→ approaches ∞ w at tips

Spanwise

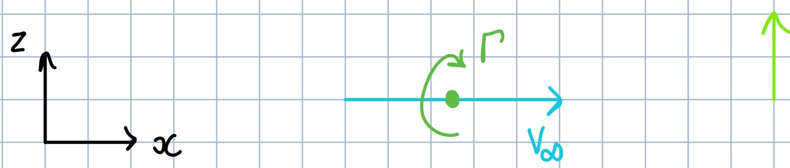


Velocity induced along wing bound vortex is a pure downwash, proportional to Γ

Effects of Trailing / Tip Vortices :

Considering local flow over an element of bound vortex :

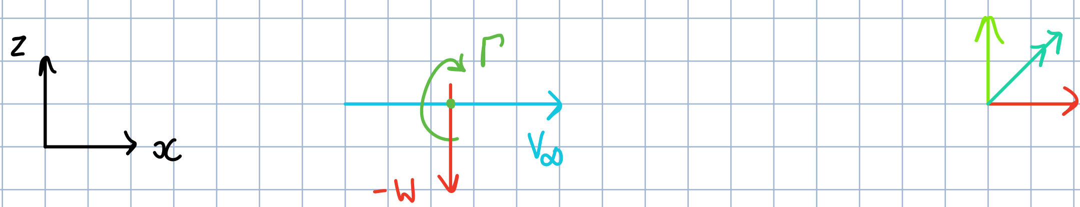
Without tip vortices



$L = \rho_\infty V_\infty \Gamma$

With tip vortices → downwash effect can be superimposed :

resultant = L_1



w is velocity in z direction
∴ downwash is -w value

$d = -\rho_\infty w \Gamma$

one comp. of drag
→ induced drag due to tip vortices

→ Presence of tip vortices = drag

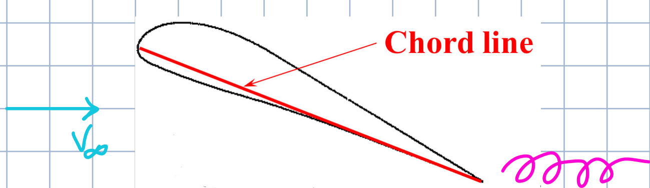
→ Downwash velocity $-w \propto$ tip vortex strength Γ

$-w \propto \Gamma \longrightarrow -w \propto L$ (as $L = \rho_\infty V_\infty \Gamma$)

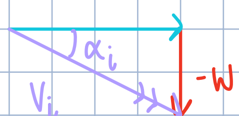
$d_i \propto \Gamma^2 \longrightarrow d_i \propto L^2$ (as $d = -\rho_\infty w \Gamma$)

induced drag (aka lift-dependent, Prandtl or vortex drag)

Alternate Interpretation of Tip Vortices :

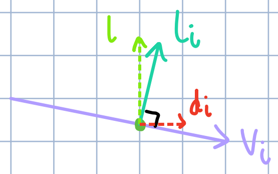


tip vortex → downwash w



Effective onset flow V_i at induced incidence α_i

→ flow can be thought of as being due to rotated freestream:



$$\boxed{L_i = \rho_\infty V_i \Gamma}$$

- Horseshoe Model provides simple demonstration of the fundamental reason for existence of induced drag.

- Can also give 1st order estimates of interactions between lifting surfaces (e.g. wing & tail, biplane, formation flight, ground effect)

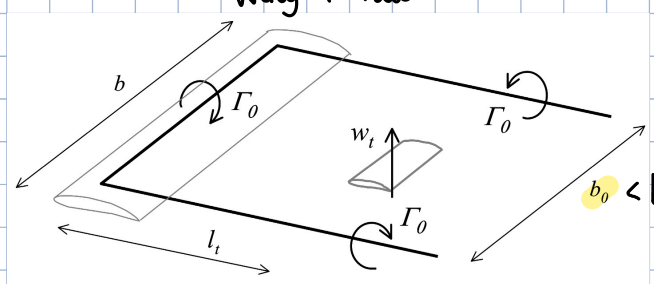
→ usually modified so total lift of horseshoe model = actual wing lift and total vortex strength is same

What's difference in modified model?

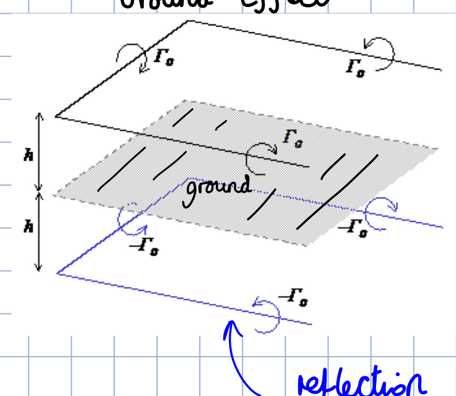
- Bound vortex doesn't cover full span → smaller ($b_0 < b$)

Example Uses of Horseshoe Model:

Wing & Tail



Ground Effect



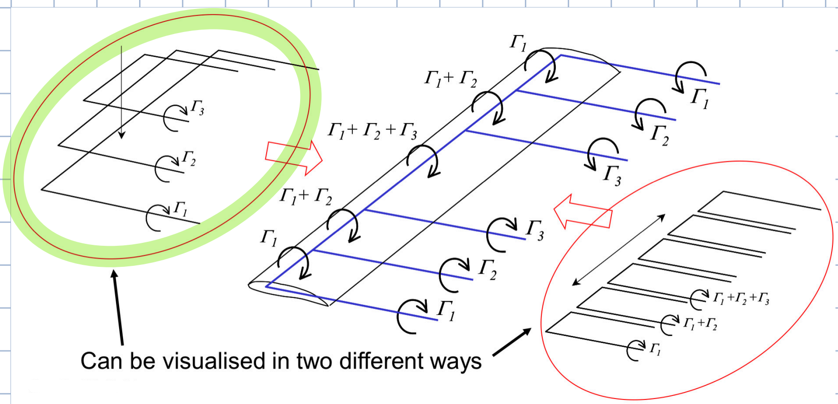
reflection gives no normal flow at ground

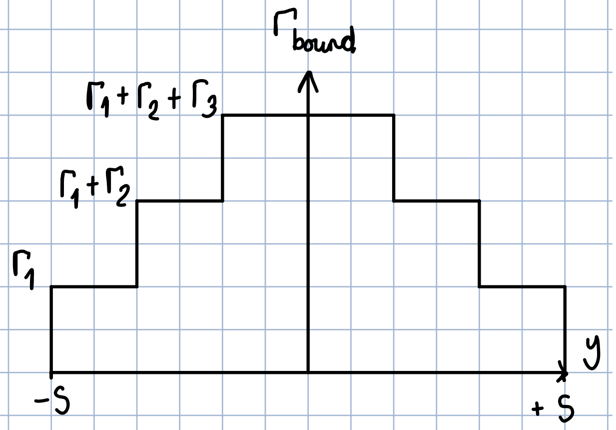
Defficiencies of Horseshoe Vortex Model:

- Predicts ∞ downwash velocities as you approach wingtips → not physical
  - ↳ tip vortices estimated too strong ∴ d_i too high
- Tip vortices only shed at wingtips → in reality vorticity shed along whole trailing edge
- Bound circulation should vary w. span (horseshoe = constant)

Classical Lifting Line Theory:

- Variation of circulation along span modelled by superimposing horseshoes of varying strength.
- Placed along $\frac{1}{4}$ chord line

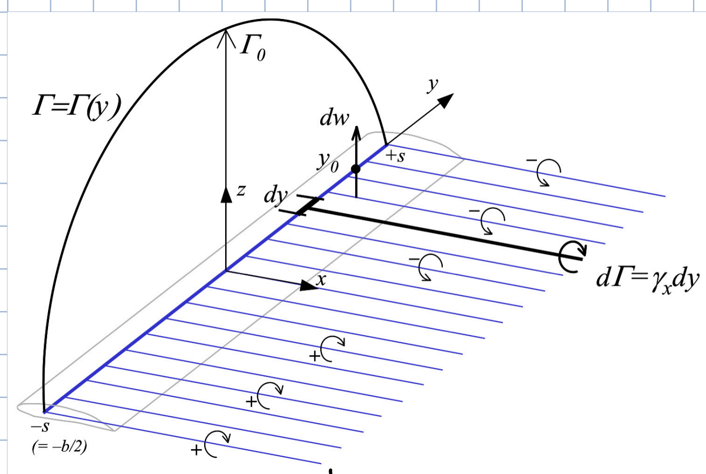




Each time the circulation jumps, another vortex shed into wake with circulation = jump

→ use infinite number of horseshoe vortices, each of strength $d\Gamma$

↳ get continuous bound circulation distribution $\Gamma(y)$ on lifting line and continuous vortex sheet shed from TE strength $\gamma_x(y)$



↳ horseshoes → ∞